

STRING THEORY : Where are we now?

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Abstract

This is a brief overview on the current status of string theory for non-specialists. The purpose is to give an aspect on the nature of string theory as a unified theory of all interactions including quantum gravity and to discuss future perspectives. Particular emphases are put on the mysteries why string theory contains gravity and why it resolves the ultraviolet problems.

1. History

It has long been recognized that the two main theoretical frameworks of present day physics, quantum theory and relativity theory, are not easily reconcilable together microscopically. Namely, the treatments of gravity using the methods of ordinary quantum field theory almost necessarily lead to the nonrenormalizable ultraviolet infinities. String theory is an attempt towards the ultimate theory which should explain all of the particle interactions and the fundamental structure of matter and space-time, by resolving the ultraviolet difficulties and the associated problems in a natural scheme where all other particle interactions can also be taken into account in a completely unified manner. In this talk, I would like to explain why string theory is promising for this direction and what is the present status of the development of the theory.

In view of the nature of this talk, it seems appropriate to start with some account of history. Please refer to the table in the next page.

The year 1998 was the 30th anniversary of string theory [1]. The first clue for string theory came from the discovery [2] of simple formula for scattering amplitudes for hadrons

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(‘strongly’ interacting particles). They satisfy a duality symmetry, called the ‘ s - t ’ duality at that time. This symmetry essentially says that we can represent the amplitudes symmetrically from the dual viewpoints of evolutions, *either* along the usual time-like direction *or* along space-like direction. Both give the same equivalent description of the physical scattering amplitudes. It soon turned out that such amplitudes are beautifully described by the quantum mechanics of relativistic strings [3]. In particular, the above s - t duality is naturally encoded in the familiar mathematical properties of Riemann surfaces, which are interpreted as the base space for quantum mechanics of strings, namely, the two-dimensional field theory (*conformal field theory*) describing the dynamics of the world sheet swept out by strings in the target space-time.

After a few years of first explosion on the establishment of string theory as the theory of hadronic interactions, it was soon understood that the theory might rather be regarded as an extension of general relativity [4] [5] [6] and gauge theories (1972~ 1977) [7] [8]. However, it took a decade for particle physicists to recognize its significance as the guide for unified theory. The main reasons for this was that, at the same period, great developments are paralleled in renormalization theory of non-Abelian gauge theories which made us possible to describe the hadronic interactions in terms of ordinary local quantum field theories. The successes prompted most particle physicists to further extensions of gauge field theories to unify all interactions including gravity. Such attempts culminated into the construction of theory of supergravity (1976~ 1980) [9] which generalized the general covariance to its supersymmetrically extended version. Actually, the most extreme of supergravity theory, $N = 8$ supergravity in four dimensional space-time could also be understood as the dimensional reduction from 10 dimensional (super) string theory.

1968 ~ 1971	Veneziano, Virasoro-Shapiro formulae String interpretation (Nambu-Goto action) (fermionic strings)
1973 ~ 1976	Relation with Yang-Mills theory and General Relativity (Einstein theory and Supergravity) (ultraviolet finiteness)
1985 ~ 1989	Classification of perturbative string theories in 10 space-time dimensions (I, IIA, IIB, $H_{SO(32)}$ and $H_{E_8 \times E_8}$)
1994 ~ present	Connection ('dualities') among perturbative string theories 'M-theory' conjecture (connection to 11D supergravity) Role of 'D-branes'

However, we soon understood that even supergravity could not resolve the ultraviolet difficulty of general relativity. This is essentially due to the fact that supergravity is not unique if we include (space-time) higher derivatives. Namely, supersymmetry is not sufficient to control the short distance space-time structure and hence the ultraviolet divergences which are inherent to all local-field-theory approaches to gravity. On the other hand, in the infrared, supersymmetry is very powerful. For instance, the classical action of N=8 supergravity is unique if we forbid higher derivatives than second in the field equation. We also understood that when one attempts to include the gauge interactions that could possibly fit in with the standard gauge models, one usually encounters various anomalies which violate the classical gauge symmetries and general covariance at the quantum level. It turned out [10] that the quantum anomalies of gauge symmetry and general covariance could be resolved in the field theories which could be regarded as the limit of supersymmetrical string-theory models.

The last observation opened up the second explosion of string theories (1985 ~ 1989), where the perturbative string theory models [1] corresponding to stable perturbative vacua in flat 10 space-time dimensions are classified into five theories, type I $SO(32)$, type IIA, IIB, heterotic $SO(32)$ and $E_8 \times E_8$. However, if one goes to lower dimensions by compactifying extra six space-time dimensions from 10 dimensions, it turns out that innumerable possibilities exist for the stable perturbative vacua. Thus the perturbative string theory has no predictive power for physics in 4 dimensions. It is also noted that

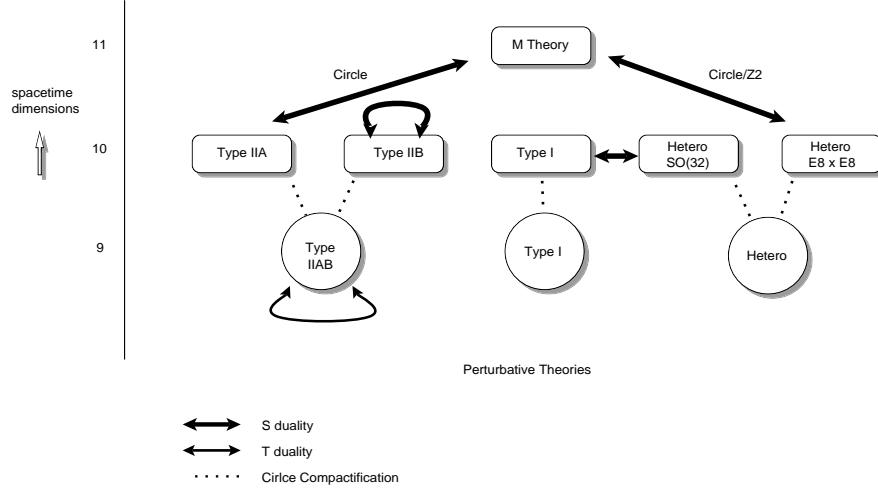


Figure 1: A schematic diagram showing various duality relations among perturbative string theories.

around from this period, the interests on string theory from the side of mathematicians arose. In particular, the conformal field theories with a variety of nontrivial compactified target spaces provided various new interplays between mathematics and physics.

During several years after the second explosion, some of the physicists have attempted to find the ways of formulating string theory in a nonperturbative fashion. For example, one such approach was to study certain toy models (called now ‘old’ matrix models [11]), which were soluble as string theory in lower dimensions, such as 0+0, 1+0, 1+1 or even ‘negative’ dimensional target space-times. It suggested some interesting hints on the structure of non-perturbative formulation, but unfortunately could not reach to spectacular successes from the original viewpoints of string unification. Around the same period, a different type of toy models became a focus of intensive studies, namely the topological field theory. Its physical significance is not clear. On the mathematical side, however, the topological field theory provided new powerful methods in certain area of algebraic topology and/or geometry.

Since around 1994 till present, we are in the third explosion of developments of string theory. This began with the improved understandings [12] on the relationship among the perturbative string vacua. In particular, we are now gaining a good grasp on the relation among the five perturbatively consistent string theories. They are connected by various duality relations which exchange the regimes of weak and strong string couplings.

The diagram above indicates the typical relationships. Here nine dimensional theories in the third line of the diagram are obtained by the dimensional reduction of ten dimensional theories by compactifying one spatial dimension into a circle of radius R . The ‘S-duality’ interchanges the strong and weak coupling regions $g_s \rightarrow 1/g_s$, while the ‘T-

duality' reverses the radius of the compactification circle $R \rightarrow 1/R$ measured in the string unit where the fundamental string length parameter is set to one[†]. A crucial assumption in this relation is the existence of 'M-theory'[13] such that it reduces to 10 dimensional type IIA (or Heterotic theory) by the dimensional reduction on a circle (or on a circle/ Z_2) with radius $R \sim g_s \ell_s$. Also we found new degrees of freedom, Dirichlet-branes [14], which are crucial building blocks to establish the above duality relation. They can be formulated as dynamical objects attached to the end points of open strings and behave as a sort of soliton-like excitations in string theory, which correspond to various monopole and instanton solutions in the low-energy field theory approximation to string theories.

In view of this duality relation, we now believe that there must exist a unified theoretical framework in which all known perturbative string theories can be derived as 'classical' solutions. In such a framework, we will perhaps be able to proceed to study the true dynamics of microscopic physics near the Planck length and thereby to give definite predictions.

2. Merits of string theory

Now, is there any evidence for believing such a promise? Or is it merely a wild fancy of string physicists? Although it must be a long way to fix this question, we can at least mention the following points as merits or achievements of present string theory.

1. Encompasses almost all past ideas towards unification of particle interactions:

The past ideas include gauge invariance, Kaluza-Klein mechanism, supersymmetry, *etc.*

2. Provides several new perspectives for understanding the dynamics of ordinary gauge field theories:

The most recent and remarkable example of this is the AdS/CFT correspondence [15], among others.

3. Provides a realistically possible and conceptually satisfying scheme of unifying all interactions including gravity:

For example, the interaction and motion become a completely unified concept in string theory, and gravity is automatically included as an intrinsic property of its mathematical structure.

4. Solves ultraviolet difficulty which is inherent to all the perturbative theories of particle theories with local interactions:

[†] Do not confuse this S-and T- duality with the s - t duality discussed before.

Previous attempts to unify gravity suffer from the ultraviolet difficulty. The removal of the ultraviolet difficulty within the usual framework of local field theory or in an extended framework allowing non-local interactions usually suffers from the violation of unitarity.

5. Provides for the first time a microscopic explanation [16] of black hole entropy in terms of quantum statistical language:

This is based upon the interpretation of the extremal and near extremal black holes in terms of Dirichlet-branes. The Dirichlet branes are the key for the most recent developments of string theory.

The importance of resolving the ultraviolet problems with gravity being included can never be overemphasized. For example, if we try to compute the entropy in the usual local field theory, we necessarily encounter ultraviolet infinities, since the Newton constant is always infinitely renormalized. Not only that, the renormalization also forces us to introduce infinitely many other dimensionful constants to write down the microscopic theory. Remember that, in the history of quantum theory, the statistical interpretation of the entropy of black body played an indispensable role in identifying the correct microscopic degrees of freedom. We have to remind ourselves that the ultraviolet catastrophe of classical field theory 100 years ago has never been completely resolved when we take into account gravity. Certainly, string theory provided the first (and only known) promising direction toward its resolution.

3. Problems of string theory

Although string theory is really promising in this way, it is certainly true that the theory has its own problems in its present stage of developments. String ‘theory’ at present is merely a collection of rules of games for constructing scattering amplitudes (elements of S-matrix) using the various datum of conformal field theory. It is indeed an extension of the standard Feynman rules for constructing the scattering amplitudes in quantum field theory of particles based on perturbation theory. The datum for the particle Feynman rules are ‘propagators’, describing the dynamics of world lines swept out by free particles in space-time, and ‘vertices’ which describe the interaction, namely, the transformations among particles in space-time such as emission and absorption of particles. In string theory, these datum are unified into conformal field theories on world sheets. The rules are astonishingly tight, self-consistent, and most importantly they conform to crucial physical requirements for acceptable physical scattering amplitudes in general quantum theory, such as unitarity (conservation of probability). In particular, comparing with the particle Feynman rules, the string ‘Feynman rules’ achieve a complete unification of free

propagation and interactions of particles, while in particle theory they must be given independently. In other words, we would need, in general local field theory, separate principles for determining completely the particle spectrum and interactions. However, we must admit that we have not yet arrived at a satisfactory understanding on why that works so well, why that conforms to general relativity at long distances, and what the basic principles are behind the rules. Worse than that, we cannot at present give definite physical predictions from string theory, because we do not know the real nonperturbative definition of string theory. Perhaps, our goal will be envisaged in the course for resolving these mysteries of present string theory.

Therefore, the most fundamental problem in string theory at present is to explore possible directions towards its nonperturbative formulation and the principles behind the rules based on which we can confidently construct the concrete mathematical framework.

To explain the nature of such explorations, I will discuss some important mysteries, lying at the heart of physical properties of string theory rules, whose origin have not been understood completely even after the various surprising developments achieved in 30 years. I hope that by so doing I might be able to convey some of the flavors to you on the matters what we are aiming to. I will take two problems, first why string theory contains gravity, and secondly why string theory can resolve the ultraviolet infinities.

4. String to gravity

Now, in what sense, does string theory contain gravity? Everyone here knows that gravity, as Einstein taught us, is formulated as the space-time geometry based on (pseudo) Riemannian geometry of space-time. In physical terms, this amounts to formulating gravity as a field theory of space-time metric and requires to treat all particle fields as geometrical objects (sections, connections, *etc*). However, string theory in its present formulation does not require such geometrical objects at least at their starting point. Indeed, the usual formulation of string theory is done assuming just the flat space-time. Thus from the traditional point of view of dynamics, string theory, especially in its classical theory, is merely describing the motion of strings in flat space-time, and hence could not be the dynamical theory of space-time metric itself. How can string theory be the theory of gravity?

4.1 *A quantum physicists's derivation of general relativity*

To understand this, it is useful, before going directly to string theory, to take a brief digression on an elementary derivation of general relativity from a purely physical viewpoint of field theory without relying upon the Riemannian geometry.

The basic idea of quantum field theory is that all the fundamental forces of nature can be understood as a result of exchanges of quanta corresponding to each force, such as photon (electromagnetic interaction), W-Z bosons (weak interactions), gluons (strong color force). The quantum of gravity is called graviton. The field theories of these quanta are constructed following the classic example of Maxwell theory. The photon is represented by the electromagnetic field or its vector potential $A_\mu(x)$ which universally couples to electric current $j_\mu(x)$,

$$-\partial^2 A_\mu = e j_\mu, \quad \partial_\mu j_\mu = 0$$

where e is the unit of electric charge (or electrical coupling constant). Here and in what follows, we will use the Euclidean conventions for the space-time indices. Unitarity requires that the only physical components of the vector potential are the transverse ones, since otherwise the time component of the vector potential yields negative probability according to usual probabilistic interpretation of quantum theory. This leads to the gauge invariance requirement: Physical observables must be invariant under

$$A_\mu \rightarrow A_\mu + \partial_\mu \lambda \tag{4.1}$$

with $\lambda = \lambda(x)$ is an arbitrary scalar. The field equation is then modified to

$$(-\partial^2 \delta_{\mu\nu} + \partial_\mu \partial_\nu) A_\nu = e j_\mu, \tag{4.2}$$

This is a consistent field equation as long as the current is conserved. The property of the electromagnetic force is precisely explained in this framework: For example, in the lowest order approximation with respect to e , the force is computed from the expectation value of the two-point correlator $\langle (\int A_\mu j_\mu)^2 \rangle$. Masslessness corresponds to the long-range nature of Coulomb force and nondefinite sign of the charge density j_0 to the existence of both repulsive and attractive forces.

Now what is the corresponding construction for graviton? Since gravity is again a long-range force, the graviton must corresponds to a field satisfying the same massless field equation on-shell. Also it is always attractive and couples universally to mass or energy-momentum. Only candidate for the currents leading to this property of universal gravitation is the energy-momentum tensor $T_{\mu\nu}$, which is a second rank conserved tensor current.

$$T_{\mu\nu} = T_{\nu\mu}, \quad \partial_\mu T_{\mu\nu} = 0. \tag{4.3}$$

Thus the potential must also be a second-rank and symmetric tensor $h_{\mu\nu}$.

$$-\partial^2 h_{\mu\nu} = 2\kappa^2 T_{\mu\nu}$$

where κ^2 is proportional to Newton's gravitational constant. Unitarity again requires that only transverse components survive, which is ensured if the gauge invariance is assumed under

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_\mu \lambda_\nu + \partial_\nu \lambda_\mu. \quad (4.4)$$

This leads to the field equation

$$-\partial^2 h_{\mu\lambda} - \partial_\mu \partial_\lambda h_\nu^\nu + \partial_\nu \partial_\lambda h_\mu^\nu + \partial_\mu \partial_\nu h_\lambda^\nu - \eta_{\mu\lambda}(-\partial^2 h_\nu^\nu + \partial_\alpha \partial_\beta h^{\alpha\beta}) = 2\kappa^2 T_{\mu\nu}. \quad (4.5)$$

This form is unique if we assume that the left hand side is of second order in the space-time derivatives and that the equation is Lorentz invariant. However, the result combined with the conservation law (4.3) of the energy-momentum tensor is not completely consistent. The reason is that the graviton itself has nonzero energy. Hence, the exchange of the graviton induces the change of energy of matter and leads to the violation of the conservation law when we only take into account the matter energies. But if we take into account the energy and momentum of graviton to recover the conservation law, we have to include the graviton field to its second order in the energy-momentum tensor by adding the contribution of $h_{\mu\nu}$, $T_{\mu\nu} = T_{\mu\nu}^{matter} + T_{\mu\nu}^h$. This in turn modifies the equation of motion of the graviton itself and the gauge transformation too. The modification of graviton field equation leads to a further modification of the conservation law of the energy-momentum in the next order. And hence we have to again modify the energy-momentum tensor. This process continues to infinite order in the graviton field. The field equation thus becomes non-polynomial with respect to the graviton field.

It is an old common knowledge [17] that the final result is nothing but the formal power series expansion obtained from the Einstein field equation $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa^2 T_{\mu\nu}$ by introducing the graviton field $h_{\mu\nu}$ as

$$g_{\mu\nu} = \delta_{\mu\nu} + \kappa h_{\mu\nu} \quad (4.6)$$

and the modified gauge transformation is equivalent to the transformation law of the metric $g_{\mu\nu}$ under the general coordinate transformation. To summarize, what we have seen is that, under a few reasonable assumptions, the requirement of getting consistent field theory for graviton could lead to general relativity even if we did not know the Riemannian geometry at all. It should be noted that the requirement of unitarity (or gauge invariance) and also the tacit assumption that the field equation contains the space-time derivatives only to second order are the essential assumptions for the above derivation.

We have emphasized that there is a big conflict between quantum theory and general relativity, since it leads to ultraviolet difficulties. However, it does not mean that both frameworks are *fundamentally* contradictory to each other. In fact, at large distances, the situation is quite contrary. For an example, an old story in the famous debates between

Einstein and Bohr shows that the quantum mechanical uncertainty relation $\Delta E \Delta t \geq h$ is consistent with the equivalence principle when it applied to the measurement of weights in weak gravitational fields. Their mutual contradiction is manifested only at sufficiently short distances near the Planck length, where the quantum gravitational effects become of the same order as other nongravitational effects. The lesson we learn from this seemingly elementary discussion is that the geometric formulation of gravity at large distances could well be a natural consequence of some well-defined microscopic theory of gravity which could possibly be based on new principles entirely different from the ordinary Riemannian geometry, but be perfectly consistent with the fundamental principles of both general relativity and quantum theory in the sufficiently large distance region. This, I think, is a suggestive lesson in pursuing the unification of geometry and quantum theory. From the viewpoint of quantum field theory, genuine observable quantities are only S-matrix elements. Even the geometry itself must ultimately be constructed from S-matrix, if we emphasize the operational aspect of any physical theory.

4.2 Why does string theory contain General Relativity ?

Let us now discuss why and how string theory contains general relativity. First we briefly summarize the string Feynman rules.

String Feynman Rules

1. string world sheet = Riemann surface :

particle quantum mechanics

\downarrow

two-dimensional (super)conformal field theory

2. S-matrix : defined by the following path integral

$\Sigma = \{\text{Riemann surfaces}\} \rightarrow M = (\text{super}) \text{ space-time}$

$$\sum_{\Sigma \in \mathcal{M}} \int_{\mathcal{M}} [dx d\psi] g_s^{-\chi(\Sigma)} \exp \left[-\frac{1}{4\pi\alpha'} \int_{\Sigma} d^2\xi L(x, \partial_{\xi}x, \psi, \partial_{\xi}\psi, \dots) \right] \prod_i \int d^2\xi_i V_i(\xi_i)$$

3. world-sheet lagrangian

$$L = g_{\mu\nu}(x) \partial_{\bar{z}}x^{\mu} \partial_zx^{\nu} + \dots$$

4. (string length)² = $\alpha' = \ell_s^2$

5. string coupling = g_s

$$\chi(\Sigma) = 2 - 2g - h - e$$

$g = \#$ of handles, $h = \#$ of holes, $e = \#$ of external lines (punctures)

Here x is the bosonic coordinate $x^\mu(\xi)$ and $\psi(\xi)$ collectively represents all the other world-sheet variables, and $z = \xi_1 + i\xi_2, \bar{z} = \xi_1 - i\xi_2$ are the homomorphic coordinates of a Riemann surface. $g_{\mu\nu}$ is the metric of the target space-time, which is flat $g_{\mu\nu} = \delta_{\mu\nu}$ in 10 dimensional perturbation theory but is curved when we consider a compactified space-time. The integral $\int_{\mathcal{M}} [dx d\psi]$ symbolizes the path-integral on a Riemann surface corresponding to a given topology of the world-sheet. The summation symbol $\sum_{\Sigma \rightarrow \mathcal{M}}$ means that the sum over all nonequivalent Riemann surfaces are made. The weight factor $g_s^{-\chi(\Sigma)}$ can actually be absorbed into the world-sheet Lagrangian by introducing the two-dimensional Einstein term $\frac{1}{4\pi} \int d^2\xi R^{(2)} \sqrt{g^{(2)}} \phi(x)$ coupled to the external dilaton field ϕ by the constant shift of the dilaton $\phi \rightarrow \phi + \log g_s$, where $g_{ab}^{(2)}$ is the intrinsic metric for two-dimensional world-sheet. The summation over all topologies and over the moduli spaces of Riemann surfaces just fits to the requirement of unitarity of quantum theory. Namely, the singularities caused at the boundaries of the moduli space give the correct physical singularities of unitary amplitudes. The initial and final asymptotic states are represented by the product of vertex operators $V_i(\xi_i)$ which have one-to-one correspondence with the physical states which manifest themselves as the singularities at the boundaries of the moduli space of the Riemann surfaces.

From the viewpoint of two-dimensional field theory, the rules are characterized by the local conformal invariance (or Weyl) invariance. Namely, the theory is invariant under the Weyl transformation (and its supersymmetrical generalization) $g_{ab}^{(2)}(\xi) \rightarrow \rho(\xi) g_{ab}^{(2)}(\xi)$ of the intrinsic metric of the world sheet. Although the two-dimensional Einstein term apparently coupled with dilaton violates this symmetry, it actually is needed to cancel a quantum anomaly of the Weyl transformation associated with the vertex operator for dilaton, which is a massless scalar excitation contained inevitably in string theory and accompanied by the graviton. The requirement of local conformal invariance is the most crucial property of the string Feynman rules and leads to the following properties of the string S-matrix.

1. Complete unification of motion and interaction:

The particle spectrum and interactions are determined simultaneously, since locally there is no distinction between motion and interaction on the Riemann surface.

2. Existence of massless spin 2 state = graviton :

From the viewpoint of particle spectrum, this is schematically explained as follows. For simplicity, we explain it in the bosonic string theory. The case of theories with fermionic degrees of freedom is basically the same. The world sheet-conformal invariance leads to one constraint and one gauge freedom in each space-time dimensions for the bosonic coordinate for the left and right moving modes (namely, holomorphic and anti-holomorphic modes) separately.

$$\begin{aligned} x^\mu(\xi) &= x^\mu(z) + \bar{x}^\mu(\bar{z}) \\ L(z) &= (\partial_z x)^2 + \cdots = 0, \quad \bar{L}(\bar{z}) = (\partial_{\bar{z}} x)^2 + \cdots = 0 \\ z &\rightarrow f(z), \quad \bar{z} \rightarrow \bar{f}(\bar{z}) \end{aligned}$$

This reduces the number of physical modes by two in each direction and hence the first orbital excitations are decomposed into the irreducible components with respect to the rotation group $O(D-2)$ of transverse directions as

$$D^2 \rightarrow (D-2)^2 = \frac{D(D-3)}{2} \oplus \frac{(D-2)(D-3)}{2} \oplus 1.$$

In the language of relativistic quantum field theory they are represented by the massless symmetric tensor (graviton) $h_{\mu\nu}$, massless antisymmetric tensor $B_{\mu\nu}$ and massless scalar (dilaton) ϕ with gauge invariance under $\delta h_{\mu\nu} = \partial_\mu \lambda_\nu + \partial_\nu \lambda_\mu$, $\delta B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and $\delta \phi = c$ with c being a constant, respectively. Actually, because of the above mentioned anomaly, the gauge transformation of ϕ must be associated with the change of string coupling $\delta g_s = c g_s$.

3. Background independence :

This comes about because there is a 1 to 1 correspondence between vertex operators and physical states of strings. In particular, deformation of background metric $g_{\mu\nu}$ is absorbed by the condensation of graviton modes graviton vertex operator : $h_{\mu\nu}(x) \partial_{\bar{z}} x^\mu \partial_z x^\nu$ for $\partial^2 h_{\mu\nu} = 0 = \partial_\mu h_{\mu\nu}$.

This has the correct gauge symmetry property on shell. Namely the gauge variation

$$\delta h_{\mu\nu}(x) \partial_{\bar{z}} x^\mu \partial_z x^\nu = \partial_z \lambda_\mu \partial_{\bar{z}} x^\mu + \partial_{\bar{z}} \lambda_\mu \partial_z x^\mu$$

is a total derivative, due to the world-sheet equation of motion $\partial_z \partial_{\bar{z}} x = 0$. Similarly the gauge symmetry is also valid for the antisymmetric tensor $B_{\mu\nu}$ too.

More generally, all possible deformations of the world sheet lagrangian (including the boundary conditions in the case of open strings) are absorbed by the condensation of various modes of string theory (if D-branes are taken into account).

Thus string theory automatically is a quantum-dynamical theory of space-time, although it has not been constructed as such. Consistency with unitarity and Lorentz invariance ensures that the theory is bound to be consistent with General Relativity at long-distance regime, provided that the low-energy limit is described by local field theory. The last point is connected with the next item. We have used the terminology ‘background’ independence. This is *not* meant that the present string theory is already formulated in a completely background independent manner. But the structure of the theory locally in an infinitesimal neighborhood of the theory space suggests that a truly background independent formulation should be possible.

4. The dynamics of strings is local with respect to world sheet:

In particular, the factorization property of the world sheet is satisfied. Namely, if we pinch off some cycles of the Riemann surfaces, the surfaces in general change topology or factorize into two disjoint surfaces. Since the dynamics of the world sheet is local, the dynamics also faithfully reflect the change of topology and disjointing of the surfaces. This implies that in the limit where the string length parameter $\ell_s = \sqrt{\alpha'}$ vanishes, the string Feynman rules smoothly reduce to those of ordinary particle theories. Combining with some dimensional consideration, this leads to the conclusion that the low-energy limit of the rules must be described by the local field theory with only two powers with respect to derivatives in any space-time dimensions. The explicit computations of scattering amplitudes involving gravitons have been carried out long time ago and provided confirmations of the general arguments [4] [5].

This is how string theory in general contains gravity (supergravity in supersymmetrical cases). Most of you perhaps now understand why I have called the existence of gravity a *mystery* of string theory. Although we understood how gravity is contained at a ‘phenomenological’ level, we do not have a ‘theoretical’ explanation. Why do the Weyl invariant string Feynman rules lead to gravity and other gauge forces at long-distances? There must be some fundamental mathematical formulation which will give ‘geometrical’ explanations on this surprising phenomenon. That would provide proper principles based on which string theory is reformulated in a completely nonperturbative and background independent fashion.

5. Resolution of UV divergencies

The next mystery I would like to discuss is why the ultraviolet difficulties are resolved [18] in string theory. This again is regarded as a consequence of the conformal invariance of the string Feynman rules. Basically, there is no ultraviolet region in those rules, since

the singularities of the moduli space of Riemann surfaces only reside on its boundaries. However, as we have already mentioned, the singularities of the scattering amplitudes caused at the boundaries of the moduli space are mostly physical ones required by unitarity. In contrast to this, the particle world-lines have extra singularities which do not corresponds to the boundary of the moduli space of Riemann surfaces. Those are points where the proper time of propagation of particles vanishes. For any finite string length parameter ℓ_s , those singularities are resolved in the Riemann surface because of the conformal invariance. A short time propagation with respect to some direction is actually a long-distance propagation of strings in another direction.

Actually, there is a danger at the boundary of the moduli space that some additional singularities may arise which are not compatible with unitarity. This is the possible problem of tachyon divergencies. If the theory contains tachyons, they yield exponential divergences associated with infinitely long proper time. That would be an indication of the instability of the vacuum one starts with in formulating the Feynman rules. This is precisely where the supersymmetry in space-time plays a proper role. If the supersymmetry is realized linearly, there can be no tachyonic excitation. As emphasized already, the supersymmetry is not powerful enough to control the ultraviolet divergences. Its proper position in string theory is rather in the infrared region to ensure the stability of perturbative vacua.

Now we have understood that both the existence of gravity and the resolution of ultraviolet difficulties usually associated with gravity in the ordinary framework of local field theory rely upon the conformal invariance of the string Feynman rules. This strongly suggests that there is some geometrical meaning for the world-sheet conformal invariance. From the point of view of two-dimensional field theories in general, choosing the conformal invariant field theories amounts to considering only a very special class of 2d field theories. In terms of the language of renormalization group, we are considering the fixed-point theories. This reminds us an analogy with the experience which physicists have had in early quantum theory in the beginning of the 20th century. To explain the spectrum of hydrogen atom in terms of classical Newtonian mechanics, we have to choose only special orbits for an electron circulating around the nucleus by imposing the Bohr-Sommerfeld condition. The latter condition is characterized by adiabatic invariance. It is tempting to compare this situation with the choice of fixed point theories characterized by conformal invariance in string theory. Then, just like there is the uncertainty relation behind the Bohr-Sommerfeld condition, it is natural to suppose the existence of some characteristic new relation by which the old theory is limited and the structure of new theory is signified. In this sense, the relation should contain the string length parameter α' which is the only free (but fundamental) parameter of string theory. Remember that the string coupling g_s cannot be regarded as fundamental, since it can be absorbed by a constant shift of the

dilaton field ϕ .

There is a very simple candidate relation which nicely fits to the above expectations. That is called the space-time uncertainty relation [19] [20]

$$\Delta T \Delta X \gtrsim \ell_s^2 \quad (5.7)$$

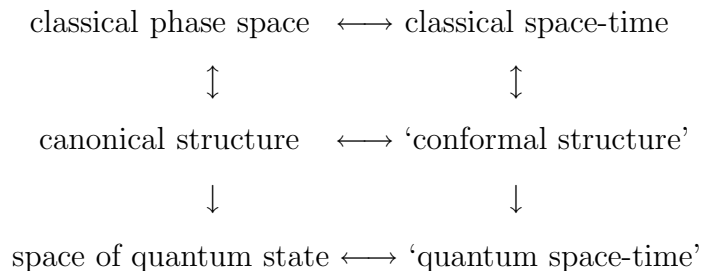
between an invariant characteristic length scale ΔT measured in the time-like direction and an invariant characteristic length ΔX measured in the space-like direction. This relation faithfully represents the manner by which the ultraviolet divergencies are eliminated in the string Feynman rules. Moreover, it turns out that the relation gives nice qualitative explanations of the properties of the interaction of D-branes. For example, the relation leads to the prediction of the characteristic distance probed by the scattering of D-particles as given by $\Delta X \sim g_s^{1/3} \ell_s$ [21] which is supposed to be the characteristic distance scale of ‘M-theory’. Since I have a plan of publishing a separate article in which the space-time uncertainty relation is reviewed extensively and developed further, I do not describe further details on this relation here. For previous reviews, see [22].

6. Future of string theory

I hope that the foregoing discussions were of some help to understand the nature of string theory and its mysterious which must be reformulated more appropriately in the future for the construction of really nonperturbative unified theory. Let me finally provide some personal viewpoints on the future development of string theory. It seems to me that further clarifications of the mysteries emphasized above are important in exploring the unified theory of geometry and quantum theory, which must necessarily be the unified theory of matter and space-time. Let us recall them again using a slightly different way of looking.

1. Formulation of the space-time uncertainty relation

The space-time uncertainty relation is still only a qualitative characterization without firm mathematical formulation. For example, we do not know the way for precisely defining the quantities $\Delta T, \Delta X$. Its original derivation [20] was based on the theory of general conformal invariants known as ‘extremal length’, but that is clearly insufficient for our present purpose since the Riemann surface is the perturbative concept. We are aiming the nonperturbative definition of the theory and therefore must find some reinterpretation of conformal invariance which does not require perturbation theory. The following analogy with the phase space of classical dynamics and its quantum version might be of some help in pursuing along this direction:



The space-time uncertainty relation can be interpreted as an analog of the ordinary Heisenberg uncertainty relation in the transition from the second line to the third line of the above diagram.

This analogy obviously suggests us to try an algebraic characterization of the space-time uncertainty relation by elevating the space-time coordinates into some operator algebra. For such an attempt, I would like refer the reader to [23] .

2. A new way of looking at gravity in string theory

The advent of D-branes has provided a new data for considering the question of why gravity is contained in string theory. The low-energy or low-velocity behavior of D-branes and their interactions are described by the Yang-Mills type matrix models with maximal supersymmetries. Based on this, an interesting conjecture for the possible formulation of M-theory has been proposed in [24] [25]. If this conjecture is true, the mystery concerning the existence of gravity is further enhanced. We cannot find any remnant of gauge invariance associated with graviton in the Yang-Mills degrees of freedom only, since the graviton in this case only appears as the t channel effect. In the s channel, there is no graviton which directly corresponds to the one exchanged in the t channel, at least apparently. On the other hand, in matrix-theory interpretation [24] [25], we have the Kaluza-Klein excitation of graviton in the s channel. Thus, if we interpret the theory as being defined in 11 dimensional space-time as M-theory, the phenomenon of reproducing gravity only using the Yang-Mills degrees of freedom is not inconsistent with general properties of string theory. Indeed, explicit computations shows that we can simulate the graviton exchanges including its first non-linear effect by using supersymmetric Yang-Mills theories. There remains many puzzles to be clarified further in the general matrix model approaches. For fuller discussions, I would like to invite the readers to our original papers [26], or to a previous review [27] and the references therein.

3. Deeper understanding on the correspondence between classical supergravity and string theory

If the general structure of string theory as we understand at present is basically justified for the ultimate unification, the new theory is characterized by one and only one new constant, string length ℓ_s . In other words, if we take the long distance limit $\ell_s \rightarrow 0$, we must be able to recover the standard quantum theory of gauge fields and classical general relativity. The former is characterized by the Planck constant \hbar , while the latter is by the Newton constant G_N . In particular, the Newton constant remains finite in the limit of going to classical physics $\hbar \rightarrow 0$. Therefore it is natural to adopt the unit where $G_N = 1$. In this unit, the dimension of the Planck constant itself is equal to the square of length. Thus we must have

$$\hbar = k\ell_s^2$$

where k is a numerical constant. In string theory, the constant k is in principle calculable in terms of the expectation value of dilaton $k = k(\phi)$. Thus in the presence of gravity, the transition from classical physics to quantum physics is equivalent to the introduction of string length. In other words, the quantization is necessarily the quantization of space-time. This line of arguments further strengthens the point of view emphasized above and suggests the importance of establishing some definite ‘correspondence principle’ between classical physics and string theory.

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